A New Monetary Policy Tool: The Real Neutral Rate Yield Curve for Canada

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ONLINE APPENDIX

To begin, we start with a basic time-varying parameter vector autoregression (TVP-VAR) that looks quite similar to that of a standard VAR,

$$y_t = \beta_{0,t} + \sum_{j=1}^{p} \beta_{j,t}\, y_{t-j} + v_t$$

where y_t is an $n \times 1$ vector of endogenous variables: real GDP growth, core inflation, and different versions of the real rate, defined as the nominal rate minus inflation expectations, which are formed through a four-quarter moving average of inflation rates and, as a secondary check, the four-quarter moving average of previous inflation and the target rate. $\beta_{0,t}$ is a $n \times 1$ vector of the intercept terms, and all $\beta_{j,t}$ are $n \times n$ matrices of coefficients for each lag, j, where we employ two lags (i.e. $p = 2$). Unlike a standard VAR, we have time subscripts on the coefficients that allow them to change over time. The number of variables in the system is n. We define the TVP-VAR more compactly by first collecting lagged variables in a matrix that we define as $X_t' = I \otimes (1, y_{t-1}', \ldots, y_{t-j}')$. Here $\otimes$ denotes the Kronecker product. We collect the coefficients and define them as $\theta_t = vec\,([\beta_{0,t}\, \beta_{1,t}\, \beta_{j,t}\,]')$. We can now write the TVP-VAR as

$$y_t = X_t'\theta_t + e_t.$$

The coefficient vector θ_t, and the coefficients within, are assumed to follow a random walk as their law of motion,

$$\theta_t = \theta_{t-1} + u_t.$$

u_t and e_t are independent. Further, each u_t is independent for each coefficient set within θ_t. We estimate the model with Bayesian techniques following Primiceri (2005) and Del Negro and Primiceri (2015). Estimates of θ_t begin with a burn-in period of five years to draw initial priors (Q1 1986 through Q4 1990). Then for each t we use the iterated forecast method to obtain the neutral rate.

The iterated forecast method uses a simple step-by-step procedure. At time t we will forecast the horizon, h, from one to 20. Using two lags, the expanded form first step of the forecast, $h = 1$, is then:

$$\hat{y}_{(t+1)|t} = \beta_{0,t} + \beta_{1,t} * y_t + \beta_{2,t} * y_{t-1}.$$

$\hat{y}_{(t+1)|t}$ is the forecast for one period ahead based on the full information to time t. A hat is used to represent that the one-step ahead value is an estimation through the TVP-VAR equation. For $\beta_{1,t}$ and $\beta_{2,t}$ the 1 and 2 denote the lag order. Because we are using a time-varying parameter VAR, these coefficients are re-estimated for each time t. For the second step we would then have:

$$\hat{y}_{(t+2)|t} = \beta_{0,t} + \beta_{1,t} * \hat{y}_{(t+1)|t} + \beta_{2,t} * y_t.$$

We can see that in this equation, we would use the previously estimated one-step ahead value of our *y* vector. Note that because this is a VAR we must forecast all three variables together, although we are only interested in our last ordered variable, the real rate. This will be our estimate of r*.

From the third step until we reach our endpoint, $h = 20$, we can then write the following for each h:

$$\hat{y}_{(t+h)|t} = \beta_{0,t} + \beta_{1,t} * \hat{y}_{(t+h-1)|t} + \beta_{2,t} * \hat{y}_{(t+h-2)|t},$$

for all $h > 2$. Once we have reached our endpoint, we store the value of the forecasted real rate as the estimated r* value for period t. This procedure is then repeated for each period t. The three-variable VAR and forecast procedures are repeated for each bond maturity to get each individual neutral rate making up the yield curve. With the two lags, our first neutral rate estimate is for Q3 1991, with the sample going to Q1 2024.

The choice of time variation is important for several reasons. The first is that the variance and parameters are allowed to vary over time. This variation over time is an attractive feature because it allows for changes in the economy to influence the neutral rate. The TVP-VAR with this variation allows for non-linear features of the economy to be captured in the framework and measured. Computationally, this is also a relatively fast method.

The TVP-VAR is chosen because it does not impose additional assumptions that may drive the model, like in other models including the Holston-Laubach-Williams (HLW) model (see Holston et al. 2017). Recent work has found improvements in the HLW approach, including methodology (Morley et al. 2023), correcting errors in the application of methods (Buncic 2020) and in allowing for transitory shocks in the HLW procedure (Lewis and Vazquez-Grande 2019).