Technical Appendix: Calculation of the Marginal Cost of Public Funds for a Land Transfer Tax

The LTT in Toronto has 5 tax brackets, indexed by j = 1 to 5. The key data on the LTT in 2020 are shown below:

$$\mathbf{ORIGIN} := 1$$

$$t := \begin{bmatrix} 0.005 \\ 0.010 \\ 0.015 \\ 0.020 \\ 0.025 \end{bmatrix} \quad \tau := \begin{bmatrix} 0.01 \\ 0.02 \\ 0.03 \\ 0.04 \\ 0.05 \end{bmatrix} \quad L := \begin{bmatrix} 0 \\ 55000 \\ 250000 \\ 400000 \\ 2000000 \end{bmatrix} \quad N := \begin{bmatrix} 40 \\ 34 \\ 850 \\ 29708 \\ 2106 \end{bmatrix} \quad P := \begin{bmatrix} 25919 \\ 153784 \\ 362794 \\ 867266 \\ 3002357 \end{bmatrix}$$

where:
t is the City of Toronto's LTT rates,
τ is the combined Toronto and Ontario LTT rates,
L is the lower limit of each tax bracket,
N is the number of land transfers in each bracket,
P is the average price of the property subject to the LTT in each bracket.

Let X denote the maximum value of the property that is subject to tax in bracket i.

$$X_i = L_{i+1} - L_i \qquad i := 1..4 \qquad X := \begin{bmatrix} 55000 \\ 195000 \\ 150000 \\ 1600000 \end{bmatrix}$$

The total LTT revenue collected in tax bracket j is equal to:

$$R_j = N_j \left(\sum_{i=1}^{j-1} \tau_i \cdot X_i + \tau_j \cdot \left(P_j - L_j \right) \right)$$

The burden of a small increase in the tax rate in tax bracket j is the following:

$$dB_j = \left(N_j \cdot \left(P_j - L_j \right) + X_j \cdot \sum_{i=j+1}^{5} N_i \right) \cdot d\tau_j$$

where the first term is the burden on property that is only subject to the rate in bracket j and the second term is the sum of the burdens on properties that are taxed in the higher tax brackets.

The change in LTT revenues that are collected from a small increase in the tax rate in bracket j is equal to:

$$dRT_j = \left(\left(P_j - L_j\right) \cdot N_j + \left(\frac{-\tau_j}{1+\tau_j} \right) \cdot N_j + \left(\sum_{i=1}^{j-1} \tau_i \cdot X_i \cdot N_j + \tau_j \cdot \left(P_j - L_j\right) \right) \cdot \left(\frac{dN_j}{d\tau_j} \right) + \left(\sum_{h=j+1}^{5} X_j \cdot N_h + \sum_{h=j+1}^{5} X_j \cdot \frac{dN_h}{d\tau_j} \right) \right) \cdot d\tau_j$$

where the first term is the "mechanical" increase in tax revenue in bracket j, the second term is the the change in tax revenue from the reduction in value of land transfers due to the capitalization of the LTT in property values, the third term is the change in tax revenue from the reduction in the number of land transfers in bracket j as a result of the higher tax rate, and the fourth term is the change in tax revenues in the higher tax brackets from the tax rate increase and the decline in the number of land transfers in these brackets.

Note that the semi-elasticity of the number of land transfers in bracket j with respect to the tax rate in bracket j is:

$$\varepsilon = \frac{1}{N_j} \cdot \frac{dN_j}{d\tau_j} \qquad \varepsilon := -7.00$$

The general expression for the MCF for the tax rate in bracket j is: $MCF_j = \dfrac{dB_j}{dRT_j}$

That is, the MCF is the ratio of the burden of a small tax rate increase to the additional revenue raised from the tax rate increase. In particular, the equation for the MCF in the top tax bracket is shown below:

$$MCF_5 := \frac{\left(1 - \frac{L_5}{P_5}\right)}{\left(1 - \frac{L_5}{P_5}\right) + \left(\frac{-\tau_5}{1+\tau_5} \right) + \left(\sum_{h=1}^{4} \frac{\tau_h \cdot X_h}{P_5} + \tau_5 \cdot \left(1 - \frac{L_5}{P_5}\right) \right) \cdot \varepsilon} \qquad MCF_5 = -480.21$$

The MCF in the top tax bracket is negative because the denominator in the above equation is negative, indicating that the combined tax LTT revenues of the City of Toronto and the Government of Ontario declines with a small increase in the top LTT rate of 5 percent.

To calculate the LTT from Toronto's perspective, we use the Toronto's LTT rates instead of the combined Toronto and Ontario tax rates, MCFto:

$$MCFto_5 := \frac{\left(1 - \frac{L_5}{P_5}\right)}{\left(1 - \frac{L_5}{P_5}\right) + \left(\frac{-t_5}{1+t_5} \right) + \left(\sum_{h=1}^{4} \frac{t_h \cdot X_h}{P_5} + t_5 \cdot \left(1 - \frac{L_5}{P_5}\right) \right) \cdot \varepsilon} \qquad MCFto_5 = 2.011$$