Appendices to Inflation after the Crisis – What's the Story?

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Appendix A

The New Keynesian Phillips curve (NKPC) equation is defined as follows:

$$\pi_t = \alpha_1 + \beta_1 E_t \pi_{t+1} + \kappa_1 x_t + \mu_{1t} \quad (1)$$

where π_t represents inflation, $E_t\pi_{t+1}$ is expected future inflation, x_t is the output gap, β and κ are positive parameters, and μ_{1t} is a stochastic shock affecting inflation.

Next, we insert a puzzle dummy in equation (1) covering the period between 2009Q3 and 2013Q4. The resulting equation is defined as follows:

$$\pi_t = \alpha_2 + \beta_2 E_t \pi_{t+1} + \kappa_2 x_t + \gamma_1 Dpc_t + \delta_1 E_t \pi_{t+1} \times Dpc_t + \theta_1 x_t \times Dpc_t + \mu_{2t} \quad (2)$$

where $Dpuzzle_t$ is the puzzle dummy, δ is the impact of the expected future inflation during the puzzle periods, and θ is the impact of the output gap during the puzzle periods.

To test different candidates the puzzle dummy might be proxying for, Friedrich reruns equation (2), replacing the puzzle dummy variable one by one with the potential explanatory variables (V_t):

$$\pi_t = \alpha_3 + \beta_3 E_t \pi_{t+1} + \kappa_3 x_t + \gamma_2 V_t + \delta_2 E_t \pi_{t+1} \times V_t + \theta_2 x_t \times V_t + \mu_{3t} \quad (3)$$

He then ranks them by those that yield the lowest mean squared error (MSE) over the full sample period. This ordering allows him to augment the Baseline NKPC specification (equation 1) with the explanatory variable with the lowest MSE. He repeats the procedure with the remaining explanatory variables. The decision to stop adding variables is based on a view that the current equation explains "an overwhelming share" of inflation dynamics over the period of interest, with a specific focus on the puzzle periods.

Friedrich finds little value-added in the interaction terms, so his final regression is:

$$\pi_t = \alpha_4 + \beta_4 E_t \pi_{t+1} + \kappa_4 x_t + \gamma_3 E_t \pi^H_{t+1} + \gamma_4 g_t + \gamma_5 p_t + \mu_{4t} \quad (4)$$

Where π^H_{t+1} is household inflation expectations, g_t is fiscal policy stance, and p_t is global energy price inflation.

Appendix B

This table includes the list of variables we use in our analysis and their definitions.

Variable	Description	Source
Measures of Economic Slack		
Output gap-1	Output gap (Integrated framework) (%)	Bank of Canada
Output gap-2	Output gap (Extended multivariate filter) (%)	Bank of Canada
Unemployment	Unemployment rate (%)	CANSIM
Industry production growth	Industry production index, averaged to quarterly, chained 2007 dollars, YoY change	CANSIM
Unit labor cost growth	Unit labour costs by persons employed, YoY change	OECD
Labor compensation growth	Total compensation per hour worked, quarterly, YoY change	CANSIM
Fiscal Policy Stance		
Government budgetary balance	General government net lending/borrowing as percentage of GDP (expenditure based, chained dollars)	CANSIM
Commodity Prices		
Energy price	The commodity price index of energy commodities (coal, WTI oil, Brent oil, Canadian Heavy Crude Oil, Henry Hub natural gas) produced in Canada and sold in world markets, YoY percent change	Bank of Canada
Food price	The commodity price index of agriculture commodities (potatoes, cattle, hogs, wheat, barley, canola, corn) produced in Canada and sold in world markets, YoY percent change	Bank of Canada
Financial Variables		
Financial Market Uncertainty	VIX Index	CBOE
Credit growth	YoY growth rate of credit to private non-financial sector as % of GDP	BIS
Stock market Growth	YoY growth rate of the MSCI Canada Index	MSCI
Real Estate Price Growth	YoY growth rate of residential property index	BIS
Inflation Variables		
CPI - Headline	Consumer Price Index (CPI), all items excluding the effect of indirect taxes	Statistics Canada
Diffusion Indices		
Diff - Cross	Diffusion index based off Cross' unweighted methodology	Kronick (2016)
Diff - PCA	Diffusion index based off Kronick's principal components analysis methodology	Kronick (2016)
Exchange Rate		
Canadian Effective Exchange Rate	Canadian-Dollar Effective Exchange Rate Index (CERI), monthly average, Index 1992 – 100	Statistics Canada
Inflation Expectations		
Long-term nominal bond yields	Government of Canada benchmark bond yields, long term	Statistics Canada
Long-term real bond yields	Real return benchmark bond yield, long term	Statistics Canada

Appendix C

Table C1: Regression Results

	Baseline (NKPC)	Baseline with Puzzle Dummy	Baseline with Friedrich Variables	Baseline with Friedrich Variables and Diffusion Index	With Interaction Terms
Inflation Expectations	0.193**	0.162*	0.0923	0.125	-0.144
	(2.30)	(1.95)	(1.13)	(1.40)	(-0.17)
Output Gap	0.388***	0.348***	0.282***	0.239***	0.802***
	(8.77)	(8.91)	(7.19)	(5.78)	(4.80)
Puzzle Dummy		-4.821***			
		(-5.94)			
Puzzle Dummy * Output Gap		0.244***			
		(3.30)			
Puzzle Dummy * Inflation Expectations		2.248***			
		(5.62)			
Food Prices			0.00281	0.00269	0.00265
			(0.63)	(0.62)	(0.60)
Energy Prices			0.00776***	0.00761***	0.00830***
			(3.80)	(3.71)	(3.90)
Diffusion Index				0.0220**	0.00230
				(2.21)	(0.07)
Diffusion Index * Output Gap					-0.0103***
					(-3.41)
Diffusion Index * Inflation Expectations					0.00502
					(0.35)
Constant	1.585***	1.672***	1.711***	0.354	1.454
	(7.85)	(8.17)	(8.73)	(0.53)	(0.71)
Observations	95	95	95	95	95
R^2	0.475	0.556	0.560	0.581	0.595

Notes: *t* statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table C2: Root Mean-Squared Error of Potential Variables	
Stock Market Growth	0.4446
Financial Market Uncertainty	0.4877
Diffusion Index	0.5171
Real Estate Price Inflation	0.5185
Labour Compensation Growth	0.5252
Baseline	0.5267
Credit Growth	0.5269
Unit Labour Cost Growth	0.5280
Industrial Production Growth	0.5362
Note: The RMSE is that of the baseline specification with Friedrich variables and each of the listed variables added individually.	

Appendix D

More formally, we assume a series of Canadian macroeconomic variables can be modeled using the following structural form:

$$B_0 Y_t = c_0 + B_1 Y_{t-1} + \cdots B_p Y_{t-p} + \varepsilon_t,$$

where Y_t is a vector of endogenous variables. For our purposes, the vector is given by $Y_t = [p_t, f_t, x_t, g_t, E_t\pi_{t+1}, \pi_t, D_t]$, where p_t is global energy price inflation, f_t is global agricultural price inflation, x_t is the output gap, g_t is the government budget balance, $E_t\pi_{t+1}$ is inflation expectations, π_t is inflation, and D_t is the diffusion index. Therefore, B_i is a 7x7 matrix for each $i = 0,1,\ldots p$, and ε_t is a 7x1 vector of error terms or structural shocks. The decision to order the diffusion index last comes from the evidence in Figure 2 suggesting a lag in the impact of economic breadth on inflation. The ordering of other variables before inflation comes from the definition of the augmented NKPC.