Making Housing More Affordable in Canada: The Need for More Large Cities

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Online Appendix A: The Model

In this Appendix, we present the spatial housing model that supports the discussion and figures presented in the text. In this model, urban land will be treated as fixed and housing policy will relate to housing density regulation. However, the implications of the model would be identical if instead we treated housing density as fixed and allowed housing policy to correspond to changes in urban land available for construction.

Environment

- 2 cities indexed by $k = 1{,}2$ (think of our big cities as encompassed in 1, and our small to medium cities in the other),
- For each city, there is a fixed supply of land L_k that is owned by resident landowners and non-resident landowners. The fraction of land held by non-resident landowners in city k is denoted by ψ_k. The resident landowners can offer some of their land on the market, or use part of it for their own housing needs. These residents are assumed not to be mobile. The mass of resident landowners in each city is denoted by m_1 and m_2; that is, the landholding of a landowner is $\frac{L_k}{m_k}$. Resident landowners also work and receive the city wage w_k. Non-resident landowners live off the rent from their land.
- There is a mass $\bar{J}$ of non-landowners or, alternatively, renter-workers, who can decide in which city to live, how much housing to acquire, and how much to consume. J will denote the mass of renters choosing city 1, with $\bar{J} - J$ being the mass choosing city 2. Renters will choose the city that maximizes their utility.
- Goods are produced by using labour N according to $y = A_k N$, where A_k represents the level of technological knowhow in city k. A key feature of the model is that there are

agglomeration externalities in the sense that A_k increases at a macro level in the population of the city (it says nothing about its distribution). Total population in city 1 is denoted by $J + m_1$, with $A_1 = A(J + m_1)$ representing the marginal productivity of workers in city 1 as a function of its population. $A' \geq 0$ is assumed throughout, implying that more population increases productivity. We will also assume that $A(\cdot)$ is always increasing in population but is bounded above. The nature of the agglomeration externalities may be different for the two cities, so the function $A(\cdot)$ may be different between the two cities. The important aspect is that $A' \geq 0$ in both cities, and is bounded above.

- Housing is built by competitive firms that can offer housing services h, by combining building and (city-specific) land according to $h = min[\phi_k L, b]$, where ϕ is a policy parameter that governs density of land, and b represents building materials, as to have housing be a combination of land and built space. High ϕ_k allows for higher density in city k. In what follows, we will begin by focusing on the effects of a housing policy directed at increasing ϕ_1, that is, increasing housing density in the bigger city. The production of one unit of building requires one unit of goods.

To keep the presentation simple, we have not included any direct congestion externality, and we have not imposed any direct cost of increasing density ϕ. These extensions can be handled easily and tend to make increased density less attractive. In the following, we also choose function forms for utility that allow many features of the model to be solved explicitly, but more generality can be easily handled.

Renter-worker Households

To determine the choices of these households, we first need to examine their choices conditional on being in a city k. This is given by solving

$$\max_{c_k^r, h_k^r} \ln c_k^r + \theta \ln h_k^r$$

s.t

$$c_k^r + p_k h_k^r = w_k$$

where p_k is the price of housing in city k, w_k is the city wage, and c_k^r and h_k^r are the choices made by a renter household in city k. The utility from the consumption of goods is given by $\ln c$ and the utility from housing is $\theta \ln h$, where θ controls the relative utility value of housing versus goods. Individual-level labour supply is fixed and set to 1. This gives rise to the demand for housing function

$$h_k^r = \frac{\theta w_k}{(1+\theta)p_k}$$

and an indirect utility function for renter-workers equal to

$$U^r(w_k, p_k) = \ln\left(\frac{w_k}{(1+\theta)}\right) + \theta \ln\left(\frac{\theta w_k}{(1+\theta)p_k}\right) \quad (1)$$

Note that the indirect utility for worker-renters is increasing in wages and decreasing in house prices, that is, people want to live in cities with high wages and low housing costs.

Resident Landowners

These households have the same preference as renter households, the only difference is that they own some local land. Their objective is

$$\max_{c_k^o, h_k^o} \ln c_k^o + \theta \ln h_k^o$$

s.t

$$c_k^o + p_k h_k^o = w_k + p_k^L(1-\psi)\frac{L_k}{m_k}$$

where p_k^L is the price of land in city k, $(1-\psi)\frac{L_k}{m_k}$ is the amount of land held by such residents and c_k^o and h_k^o are the choices made by resident landowners. Labour supply is again fixed and set to 1. This gives rise to a demand for housing function

$$h_k^o = \frac{\theta\left(w_k + p_k^l(1-\psi)\frac{L_k}{m_k}\right)}{(1+\theta)p_k}$$

Non-resident Landowners

Non-resident landowners only consume goods, meaning their budget constraint is $c_k^n = p_k^L \psi L$. They have no decision to make.

Relationship between Price of Land P_k^L and Price of Housing P_k

Competition in the building industry (zero profit condition) implies that

$$p_k = \frac{p_k^L}{\phi} + 1$$

where p_k^L is the price of land in city k.

City Wages

Competition in the goods sector implies that $w_1 = A(m_1 + \bar{J})$ and $w_2 = A(m_2 + \bar{J} - J)$, with the property that wages increase with city size.

Determination of Equilibrium Price of Land and Housing

The price of land adjusts to equate the demand and land supply in each city. Land demand in city 1 is given by

$$J\frac{h_k^r}{\phi} + m_k\frac{h_k^r}{\phi}$$

or

$$J\frac{\theta w_k}{(1+\theta)(p_k^L + \phi_1)} + m_k\frac{\theta(w_k + P_k^L(1-\phi_k)\frac{L_k}{m_k}}{(1+\theta)(p_k^L + \phi_1)}$$

Land supply is given by

$$L_1$$

Given that the price of land cannot be negative (free disposal), the equilibrium price of land in city 1 is given by

$$p_1^L = \max\left[\frac{1+\theta}{1+\psi_1\theta}\{(J+m_1)\frac{\theta w_1}{(1+\theta)L_1} - \phi_1\},0\right],$$

and the equilibrium price of housing is

$$p_1(J+m_1) = \max\left[\frac{1+\theta}{1+\psi_1\theta}\{(J+m_1)\frac{\theta w_1}{(1+\theta)\phi_1 L_1}\},1\right],$$

We can now rewrite the utility of a renter in city 1 by substituting into equation 1 the expression for wages and the price of housing. This allows us to express the utility of renters as a function of total population in city 1 as follows

$$U^r(m_1 + J) = \ln\left(\frac{\theta A(m_1 + J)}{(1+\theta)}\right) + \theta\ln\left(\frac{\theta A(m_1 + J)}{(1+\theta)p_1(J + m_1)}\right)$$

Or, if we denote total population in city 1 as $pop_1 = m_1 + J$, this can be rewritten as

$$U^r(pop_1) = \ln\left(\frac{\theta A(pop_1)}{(1+\theta)}\right) + \theta\ln\left(\frac{\theta A(pop_1)}{(1+\theta)p_1(pop_1)}\right) \quad (2)$$

A graphical representation of Equation 2 is presented in Figure 10 in the text. Because we are assuming that $A(\cdot)$ is bounded above, the utility of renters in the city will initially increase with city size, but will eventually decrease since housing costs will continue to increase while productivity remains bounded.

Note that for fixed J, increasing ϕ_1 (or increasing L) decreases the price of housing unless the price of housing is at the floor of 1 (the building cost). So, from a partial equilibrium standpoint (no intercity mobility), increasing ϕ is always good for non-landowners/renters. This is represented in Figure 11 in the text.

For city 2 we have

$$p_2^L = \max\left[\frac{1+\theta}{1+\psi_2\theta}\{(\bar{J} - J + m_2)\frac{\theta w_2}{(1+\theta)L_2} - \phi_2\},0\right],$$

and the price of housing is

$$p_2(\bar{J} - J + m_2) = \max\left[\frac{1+\theta}{1+\psi_2\theta}\{(\bar{J} - J + m_2)\frac{\theta w_2}{(1+\theta)\phi_2 L_2}\},1\right],$$

The utility of a renter in city 2 will be given by

$$U^r(\bar{J} - J + m_2) = \ln\left(\frac{\theta A(\bar{J} - J + m_2)}{(1+\theta)}\right) + \theta\ln\left(\frac{\theta A(\bar{J} - J + m_2)}{(1+\theta)p_2(\bar{J} - J + m_2)}\right)$$

If we now normalize total population to 1, and let α represent the fraction of the population that is in city 1, we can rewrite the utility of renters in city 1 and 2 as

$$U_1^r(\alpha) = \ln A(\alpha) + \theta\ln\left(\min\left[\frac{(1+\psi_1\theta)}{(1+\theta)}\frac{\phi_1 L_1}{\alpha}, \frac{\theta A(\alpha)}{1+\theta}\right]\right)$$

and

$$U_2^r(1-\alpha) = \ln A(1-\alpha) + \theta \ln\left(\min\left[\frac{(1+\psi_2\theta)}{(1+\theta)}\frac{\phi_2 L_2}{1-\alpha}, \frac{\theta A(1-\alpha)}{1+\theta}\right]\right)$$

These two equations can therefore be represented as a function of α on the same figure. This is done in Figure 12 in the text.

Equilibrium Determination of City Size J.

City size is determined by J adjusting so that a non-landowner (renter) is indifferent between the two cities, that is,

$$U^r(w_1, p_1) = U^r(w_2, p_2)$$

where both wages and house prices are a function of J. Alternatively, this can be expressed as

$$U^r(\alpha) = U^r(1-\alpha)$$

Combining elements, the equilibrium determination of α (assuming it is interior) is implicitly defined by

$$\ln A(\alpha) + \theta \ln\left(\min\left[\frac{(1+\psi_1\theta)}{(1+\theta)}\frac{\phi_1 L_1}{\alpha}, \frac{\theta A(\alpha)}{1+\theta}\right]\right)$$
$$= \ln A(1-\alpha) + \theta \ln\left(\min\left[\frac{(1+\psi_2\theta)}{(1+\theta)}\frac{\phi_2 L_2}{1-\alpha}, \frac{\theta A(1-\alpha)}{1+\theta}\right]\right)$$

This is to say that α is determined by the crossing of the two curves presented in Figure 12 in the text.

In general, the utility of worker-renters for each city, U^r, will go through three main regimes as population increases.[1] First, there is a regime where there are no housing cost implications and the city only benefits from population growth. Second, there is a regime where housing cost effects become present, but they are still dominated by agglomeration externalities. Finally, there is a third regime where housing cost effects dominate agglomeration externalities and the utility of a non-landowner decreases as population increases. In other words, as previously noted, U^r

[1] There may be more than three regimes, but for simplicity we focus on the three-regime case.

will tend to be hump-shaped, first increasing in population and then decreasing. This is a standard configuration in spatial equilibrium models.

It is important to note that, with two cities, there are only two (interior) equilibrium configurations that are possible. The two configurations depend on the marginal benefit to non-landowners of an exogenous increase in population in each of the cities (these are $\frac{\partial U_1^r}{\partial \alpha}$ and $\frac{\partial U_2^r}{\partial (1-\alpha)}$). These marginal benefits to the population can be positive or negative depending on whether agglomeration effects dominate the cost of housing effects.

The only two possible configurations are

I) Starting from an equilibrium, the marginal benefits for non-landowners of an exogenous increase in population is negative in one city and positive in the other (this is the configuration represented in Figures 12, 13, and 14 in the text).

II) Starting from an equilibrium, the marginal benefits for non-landowners of an exogenous increase in population is negative in both cities.

Note that the marginal benefit, evaluated at an equilibrium point, cannot be positive in both cities as that would be unstable: any small increase in population in one city would start a spiral of inter-city migration toward the city with the greatest (positive) marginal benefit, and this would only stop once the configuration has changed to one of the two cases above, or once all the population in a country is contained in one city.

For a country like Canada, the case described by (I) seems most relevant. In this configuration, the population in the big city(ies) is such that further increases in population tend to make the city more expensive, in the sense that housing costs dominate the higher wages. In contrast, for the smaller city, an increase in population makes it more attractive, or at least not less attractive. Accordingly, our main focus is on this configuration.[2]

Effect of increasing density (ϕ_1) in city 1 (the big city) on welfare of worker-renters

What is the effect of increasing density in the large city (increasing ϕ_1)? Assuming we are in the configuration represented in Figure 12 in the text, an increase in ϕ_1 has the following effects. It

[2] If both cities are in regime 3, we have too much population for the capacity of the country.

leads to an increase in J (a larger big city), higher house prices in city 1, higher land prices in city 1, higher wages in city 1, and decreased utility for all non-homeowners as the housing cost effects outweigh the agglomeration gains. In city 2, we have less population, lower house prices, lower wages; everyone loses in city 2. This outcome is represented in Figure 13 in the text.[3]

How do we avoid the unintended outcomes of the increase in ϕ_1? What is necessary is to complement the increased density in city 1 with a policy that simultaneously makes the smaller city more attractive. In the confines of the model, this could be done by increasing ϕ_2.[4] Such an outcome is represented in Figure 14 in the text. In this case, the utility of worker-renters increases in both cities as land prices tend to fall. The increase in ϕ_2 needs to be sufficiently strong to counter the migration incentives induced by the increase in ϕ_1. By indirectly limiting migration into the big city, the increase in ϕ_2 is allowing the increase in ϕ_1 to reflect its partial equilibrium effects.

Online Appendix B: Caveats and limitations of the analysis

The model we discuss in the paper adapts a simple framework to make the point that, to decrease housing costs in our large cities, it is essential to improve the attractiveness of our smaller cities. This result is robust to many extensions and enrichments to the model. Nonetheless, like any model, our focus has its limitations. One particular limitation is with respect to our treatment of workers as being homogenous, with all of them benefiting equally from agglomeration. However, in reality, the degree to which different households gain by working in larger cities is highly variable. This implies that the threshold level at which housing costs outweigh agglomeration gains is likely to vary across different segments of the labour force. As a result, in addition to the

[3] Given we have not included any congestion externalities in the model, landowners in the large city would necessarily benefit by the increased density. However, if we allow for congestion externalities, and if these are sufficiently important, then even resident landowners in the big city can lose from the policy. Note that non-resident landowners in city 1 always win when density is allowed to increase in the big city.

[4] Alternatively, in a more general model it could be done by subsidizing infrastructure in the smaller city.

effects we have highlighted here, different supply-side policies can have very different effects on different types of workers. Another element we omitted from our analysis is the possibility of innovation spillovers from megacities to smaller ones. In the presence of such spillovers, the mechanisms highlighted here could be overturned to varying degrees.

Online Appendix C: Policy Options when Faced With International Migration into the Big City

A common policy question is how to respond to an inflow of international immigrants who initially cluster in big cities. To this end, we briefly discuss here the insights from our model with respect to two different policy responses. In both cases, the initial effect of the increased population in the big city is to increase the price of housing, making non-homeowners in the big city worse off. This creates an incentive for endogenous out-migration toward the smaller city. Now consider the following two policy responses. The first is to adjust housing regulation in the big city so as to try to accommodate all the new arrivals. This could for example be done by allowing much higher density. The second response is to adjust housing policy so as to simply try to maintain the original size of the big city and to favour endogenous out-migration of a size comparable to the initial exogenous in-migration. This would also require some de-regulation in housing in the big city, because the new population has made the big city less attractive relative to the small city to begin with. However, this deregulation is likely to be much less intensive.

Now consider the different outcomes implied by our model following each of these policies. In the first case, the final equilibrium looks a lot like the initial equilibrium, in that non-homeowners in both the big city and the smaller city are no better nor worse off than in the absence of the initial exogenous immigration. Under this policy, the big city is bigger and more productive, but the gains are all picked by existing landowners. The small city is not affected since the policy is aimed at fully accommodating the greater population in the big city. In the second case, the small city has grown due to the outmigration from the big city. The small city becomes more productive due to agglomeration gains. In the large city, house prices have come down due to less restrictive regulation and are not bid back up to their initial level because the smaller cities have become more attractive. So, under this second policy, non-house owners gain in both cities and house owners gain in the small city. The only potential loser is now the house owners in the big city, who were the only winners under the first policy.